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PRIMARY NUMBER

THROUGH

CONSTRUCTIVE MEASURING

A MANUAL FOR TEACHERS

BY

GERTRUDE EMMONS BIGELOW

TEACHER OF MATHEMATICS IN THE BOSTON NORMAL SCHOOL

EDITED BY

WALLACE C. BOYDEN, A.M.

HEAD MASTER BOSTON NORMAL SCHOOL

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PREFACE

THIS manual is the outgrowth of an experiment in methods of teaching arithmetic in the primary grades which has been conducted during the past three years, under the supervision of the teacher of mathematics in the Normal School, in seven school districts in Boston; namely, the Bowditch, Everett, Lyman, Martin, Mather, Prescott, and Rice districts.

The plan followed successfully in the Training School in Willimantic, Connecticut, under the direction of Miss Sarah J. Walter, formerly Teacher of Methods in the Willimantic Normal School, was adopted as the basis for the work, and modifications made to suit the conditions presented by the city schools.

A general outline of the work was placed in the hands of the teachers, and frequent conferences held, at which the purposes, methods, practical difficulties, etc., were discussed. Notwithstanding delays in securing working material, and other difficulties which have proved a handicap to progress, all interested in the experiment have expressed satisfaction with the results, and confidence in the plan of work.

In the light of the results already obtained, this plan of work is commended to the thoughtful attention of all teachers in primary grades as one likely to give the children a clear and usable knowledge of numbers, greater ability for individual thinking and independent action, an increased power of expres-

sion, and a decided gain in interest in the subject, all of which should make the later work in arithmetic advance more rapidly and be less mechanical.

Masters and teachers alike have expressed a wish to have the work continued, with an earnest request for a manual which can be placed in the hands of the teachers as a guide. The one naturally and logically to undertake this work was Miss Bigelow, a teacher of wide experience, who has herself taught a class of children by this plan throughout the three years, and who has been at the same time the able and inspiring leader in the experiment.

This little book has been prepared in response to the above request, with the hope that it may be useful to those who by their cordial coöperation have made it possible, and also to those who may wish to try the work for the first time.

THE EDITOR.

PRIMARY NUMBER

NUMBER is one of the keys with which the child unlocks some of the doors leading into the wonder room of the world. He begins to use this key very early in life, showing in his play that he recognizes a difference between one and many. Before he enters school, his life experiences have increasingly developed this number sense. The work in the kindergarten helps in the same direction, and he enters the primary school with varying degrees of development but with more or less definite ideas of number. It is the work of the primary school to continue this development: not primarily to teach number facts, though these are important and have their place, but to help the child to get acquainted with number, to develop the number sense. It is the purpose of this manual to suggest some ways of doing this.

Number is essentially a relation, a relation between magnitudes. Hence the ideas of number cannot be obtained through perception alone, although careful observation is necessary that we may obtain definite images of the things to be related. But seeing relations demands more than mere observation; these images must be compared, related one to another quantitatively in many ways that we may get ideas of number.

These truths give us some valuable suggestions for primary number work. If number is a relation between magnitudes, the child who is to get the idea of number must be put directly into contact with material. He must use this material first in getting clear images; and as the power to see relations clearly

depends upon the definiteness of the image, this material must be simple, such as can be easily visualized. Next, since relations reveal themselves through comparison, the material must be used by the child in such a way that he will compare, and as we wish to reveal to him relations of quantity, the material must permit of definite numerical comparisons. Again, since these relations must be seen repeatedly under many and varying conditions, the child must have constant opportunity for constructive work, and hence the material must be such as can be easily handled and used in building many and varied forms. In a word, the material must be in the hands of the child, and must be used by him in imaging, in comparing, and in constructing.

Measures of lengths, surfaces, and solids have been found by actual experience to have characteristics which make them desirable for this work. They give clear visual images, they have definite numerical relations, and they can be freely used in construction. They are also of interest to the child, as he is already familiar with their use through his blocks at home and through some of the gifts in the kindergarten.

The greater part of the work outlined in this manual may be called number work with measures. The other part to be discussed is counting. This is, of course, closely related to measure work — McClellan and Dewey saying that when we count we measure, and when we measure we count. Counting helps the child to get acquainted with number, and therefore has a place in primary work.

The plan of this manual is to outline the work of each of the primary grades in measure work and counting, discussing the material to be used, the work to be done, and the ways of doing it.

GRADE I

MEASURE WORK

The work of the first year in number should be very simple. It is the time to develop the number sense, not to ask for number facts. The purpose may be stated briefly thus: to help the child to become acquainted with number, first by getting clear images, and next by repeatedly discovering numerical relations through comparison and simple constructive work with measures.

The work divides itself naturally into three parts, — work in linear, square, and cubic measure. While each part will be discussed by itself, it is not intended that the work of one shall be completed before that of the next is begun. The connection will be shown as the work goes on.

Linear Measure

Material. — The material needed is a supply of lengths of from 1 to 12 inches. No material can give length alone; in order that the children may handle it, there must be three dimensions. But it is found that sticks, splints, or narrow strips of cardboard generally suggest length, and length only, to the children, as that is the most prominent characteristic. Lines drawn on the board and edges of surfaces and solids are also useful. These lengths must be very accurate, an exact number of inches from 1 to 12. The measures are taken through 12 because of the common use of the foot-rule.

General Lesson. — In seeing relations the child does not begin with the definite numerical relation of twice as long or three inches shorter, but the general relation of longer or shorter. The first lessons should help him to see relations with increasing definiteness.

The greater part of the work of the first grade would be better carried on at a number table sufficiently large for fifteen or eighteen children to gather around. Let us suppose a number of children with their teacher around such a table, ready for the first lesson. On the table are scattered the linear measures 1 to 12 inches long. The teacher directs each child to pick up a measure and place it on the table just before him, then to pick up another and place it by the first. The children are asked to tell what they notice about the measures. When the answers show that length is the important characteristic seen, the teacher suggests that measures of the same length be put together. Let each child continue sorting and matching the measures, until he has several piles, and shows by the way he works that he is beginning to recognize likenesses and differences. Thus far few statements have been required from the children, although some will doubtless have used the terms "longer" and "shorter." Now all the children are encouraged to use these terms intelligently in some such way as this: the teacher directs each child to pick up one measure, to show another longer than this, and asks different children to talk about their measures until they use the phrase readily, "This measure is longer than that." Similar work is done with the terms "shorter," "just the same," "just as long," or "equal," the child always having the measures he is comparing in his own hands, placed in such a position that the fact stated is evident.

Next may come the comparison of his own measure with another's. Each child is directed to take a measure, and the teacher holds one. She tells Roy to look at his measure and at hers, and asks him what he thinks about his measure. The reply, "I think my measure is longer than yours," should at once be tested by placing both together. (Train the child from the first to the use of *I think* when he has not proved his statement, and *This is* when it is a known truth. Train him,

also, to test his judgments, and to say orally or silently, *I am right, I am wrong.*) As this work continues, expect the children to introduce such statements as, "I think Abbie's measure is very much longer than this one," or "I think John's is only a little shorter than mine." This will show that the relations are being seen with increasing definiteness. Thus far no length has been named numerically, as "this 4-inch measure," nor any definite numerical relations seen. But soon some child may say, "I think this measure is two times as long as Harold's." This indicates that he is seeing definite numerical relations, and he should be helped in this by the study of exact lengths. Before considering that part of the work, however, a few additional suggestions for the introductory lesson are given.

Suggestions for the Introductory Work.

This general lesson may continue for several days according to the children's needs. By their quickness in sorting and matching, by their accuracy in comparing, by their ready and intelligent use of language, and by the stating of a relation numerically, they will show when they are ready for the next work. Group together those who have reached this stage of development, and let them advance.

Many and varied exercises are possible in this introductory work for general comparison; a few are suggested.

Group measures according to length.

Select measures to match others (shown by the teacher).

Find measures longer, shorter, very much longer, etc.

Children choose any measure and compare with the one shown by the teacher.

Arrange the measures in steps from the shortest to the longest.

Much of the sorting and matching can be done as seat work or busy work.

Study of Exact Lengths 1 to 12 Inches Long. — There are three distinct phases in studying the measures 1 to 12 inches long; these are imaging, comparing, and constructing. The three are intimately connected: imaging is the basis for comparing; comparing makes clearer images; constructing is helped by both the others, and in turn strengthens both in a remarkable degree. In describing the work, they will be treated separately that each may be clearly seen. Yet, in the actual work of the schoolroom, this separation is not only undesirable but also impossible. Each lesson makes use of the three, although one may be especially emphasized.

Be sure that in this work emphasis is placed upon length only.

Teaching the Inch Measure.

The work begins with the study of the 1-inch measure. The child finds the shortest measure, learns the name, selects other inch measures, tries to find things about the room which measure one inch, and tries to draw lines one inch long and tests them, until he has a fairly accurate image of an inch. Imaging is the only thing to be done in the study of the inch measure at this stage.

Illustrative Lesson in Teaching a Definite Length.

The work with the measures 2 to 12 inches long follows one plan, however the details may vary. The teaching of the 5-inch measure is chosen as an illustration of this plan of work.

Upon the number table around which the children are gathered are the measures 1 to 5 inches, with a few longer ones; on the board are drawn lines in different positions, 3 to 6 inches long; about the room are a few objects having 5 inches for one dimension. The lengths 1 to 4 inches are supposed to have been taught.

Imaging.

The teacher shows a 5-inch measure, and directs the children to find one as long. Each child's is tested and the name given. Other measures like this are found and named. Each child holds up a 5-inch measure and looks at it closely. (Teach the children to hold a linear measure in the centre, that the entire length may be seen.) He moves his finger along the length, closes his eyes, and tries to image it. With measure in hand, each child tries to find something as long and reports: "This pencil is five inches long," "This board is five inches wide," "It measures just five inches from this nail to this one," proving each statement as it is given, by measuring. Even if a child fails to find anything, he has still had practice in using the length in measuring, and has a better image of it than before. Next the children are asked to name something in the room which they think is five inches long, to pick out the 5-inch lines from those drawn on the board, and to try to draw a line five inches long. Such statements are made as: "I think that board is five inches wide," "I think the line marked *a* is five inches long." The correctness of the judgment should be immediately tested by placing the measure beside the object, and the result stated as: "I was wrong, this board is more than five inches wide," or "This line is just five inches long."

The work in imaging just described aims to give the child a definite image of a 5-inch length through visualizing, through using it in measuring, and through judging, which should always be followed by testing.

Comparing.

Each child is directed to find a 5-inch measure, and to place a 4-inch measure beside it, thus ||, and is asked to tell what he notices about them. Some of the answers to be expected are, "A 5-inch measure is longer than a 4-inch

measure," "A 4-inch measure is a little shorter than a 5-inch measure," "If I put a 1-inch measure with a 4-inch measure, I think it will make a measure 5 inches long." (Let such a statement be proved.) "I think a 4-inch measure is 1 inch shorter than a 5-inch measure," "If I should cut a 1-inch measure from a 5-inch measure, it would leave a 4-inch measure" (illustrating by placing a finger where the cutting would come). Similar work should be done in comparing the 3-, 2-, and 1-inch measures with the 5-inch measure.

After the various relations have been seen, test exercises similar to the following may be given:

Arrange the measures in steps from 1 to 5 inches. Name them. Touch the 5-inch measure; the 3-inch measure; etc. Touch the 5-inch measure; touch the one 1 inch shorter; name it; etc.

Pick up the 5-inch measure; pick up the one 3 inches shorter; etc.

Show a 4-inch measure; show one 1 inch longer; etc.

The teacher holds a measure behind her, telling the children she has a measure shorter than a 5-inch measure. John holds up a 3-inch measure and asks, "Is it as long as this one, 2 inches shorter than a 5-inch measure?"

Various exercises will suggest themselves to the teacher as she continues the work. Let them be simple and concrete, giving a constant opportunity for expression of thought not only by the use of the material but also in words.

Constructing.

Each child is directed to find a 4-inch measure and a 1-inch measure, and build. Fred reports, "I put a 4-inch measure with a 1-inch measure, and have a 5-inch measure." Each is directed to find a 3-inch measure and to put another one with

this to make a 5-inch measure. Helen reports, "I built a 5-inch measure by putting a 2-inch measure with a 3-inch measure." The children are directed to build a 5-inch measure with any two measures, and the different combinations are reported. They find the number of 1-inch measures which can be placed beside a 5-inch measure, and report.

Suggestions for the Study of Definite Lengths.

Work similar to the above in imaging, comparing, and constructing should be done with the measures 2 to 12 inches long.

In the work in imaging try to have the children get clear mental pictures, but do not stay at this stage of the work until absolutely perfect ones are obtained. Remember that each of the following steps helps the imaging.

In the work of comparing do not require all the statements from each child. Let each give the relation that he sees, whether it be simply the general relation of longer or shorter, or that numerical one which we call addition or subtraction. In some comparisons the thought of multiplication or division begins, as in comparing a 3-inch measure with a 6-inch measure, "Two 3-inch measures make a 6-inch measure," or "A 3-inch measure is half as long as the 6-inch measure." As this work of comparing is continued with the different lengths, all the number facts in addition, subtraction, multiplication, and division, from 1 to 12, may be stated by the children as truths which they have had the pleasure of discovering. Do not expect these facts to be memorized at present.

The work in constructing will again emphasize the number facts, and give many opportunities for their repeated use. Give directed work in construction, but encourage original work also.

Study of the Foot-rule. — The time for introducing the foot-rule depends upon the class. When the children show that

they know the shorter lengths — perhaps 1 to 6 inches — fairly accurately, and that the testing of longer ones by the use of the inch measure is becoming cumbersome, the ruler may be given them. The foot length should be presented as a whole, as should all the measures. To do this satisfactorily a supply of lengths one foot long, not marked off in inches, is desirable. The first work is identical with that described in the study of the 5-inch measure. Let the length be named one foot, not 12 inches.

The next work shows the foot-rule as a number of inches, and helps the children to use it in measuring various lengths. For this lesson let each child around the table be supplied with a ruler 12 inches long; one having few divisions, with the figures 1 to 12 plainly marking the inches, being desirable. Place upon the table a number of the lengths which have been studied thus far.

Each child is told to find a 1-inch measure and to place it beside his ruler. His attention is directed to the mark on the ruler at the end of the measure and to the figure 1 beside it. Another inch measure is placed by the first, and the mark and figure at the end of the measure thus built are noted. The two inch-measures are taken away and a 2-inch measure placed, the mark and figure being noted as before. This is continued until the longest measure the class knows has been built. The teacher shows a 4-inch measure, and each child finds one as long. After each is tested and named — a 4-inch measure — it is placed beside the ruler and the mark and figure noted as before. This is continued with measures whose length are known until the children become somewhat acquainted with the marks and figures, and begin to realize that lengths may be measured with the foot rule. After this the ruler should be constantly used in measuring length.

Seat Work. — Some of the exercises in seat work in connection with linear measure are the following:

Sorting and matching.
Arranging in steps.
Cutting strips of paper of definite lengths, using measures.
Drawing on paper or blackboard lines of given length, using measure or ruler.
Building measures with inch sticks as long as those drawn by the teacher.
Building measures with colored sticks, showing combinations, as a measure 7 inches long built with five red inch sticks and two blue ones; or with a yellow 4-inch stick and a white 3-inch one.

Surface Measure

Material. — In surface, as in linear measure, all material must have the third dimension in order to be handled. But it is usually found that in the tablets used the thickness is so slight, that surface is what the child sees. In this manual the word "surface" used in connection with tablets refers simply to the face of the tablet apparent to the child; namely, to the upper or top face of the tablet before him. Since in using tablets the child sees surface, the phrase "a tablet containing 6 square inches," means to him "a surface containing 6 square inches," and is thus used in this manual. This use of the term "tablet" is for the sake of simplicity in language. If the teacher understands this thoroughly herself, recognizing that a tablet is a solid, but using it in such a way that the child sees surface and touches surface, there will be no difficulty.

The material needed is a supply of rectangular cardboard or paper tablets, whose dimensions are an exact number of inches from 1 to 12, and whose contents do not exceed 12 square inches. They must be cut with great accuracy in order to give satisfactory results. These are the tablets:

$1'' \times 1''$ to $12'' \times 1''$
$1'' \times 2''$ to $6'' \times 2''$
$1'' \times 3''$ to $4'' \times 3''$
$1'' \times 4''$ to $3'' \times 4''$
$1'' \times 5''$ to $2'' \times 5''$
$1'' \times 6''$ to $2'' \times 6''$

Rectangles enclosed on the blackboard and faces of solids are also useful.

General Lesson. — The introductory or general lesson in surface measure aims to give the child the idea of surface as something which can be touched, having two dimensions, — length and width. The children are gathered around the table upon which the tablets are placed. Each child takes one, moves his hand over the surface, hears and uses the name, and shows the surface of tablets and other objects, making such statements as: "I am touching the surface of a card," "This is the surface of the table." The teacher shows a tablet $6'' \times 1''$, and asks the children to find one of the same size. Each child's is tested, and the length of the tablet is noticed, the children moving their fingers along one edge. From their study of lengths some child will say that this tablet is 6 inches long. The teacher shows a tablet $6'' \times 2''$, and asks if it is like the first in any respect. When the fact is given that it is 6 inches long, it may be placed upon the $6'' \times 1''$ tablet, and the difference seen. This will help to show that we must know something besides length about a surface; we must know its width as well. Two things may then be told of each surface, as "This tablet is 6 inches long and 1 inch wide," tracing dimensions as named. Many different surfaces may be described in this way by the children, rulers being used to test lengths when necessary. The teacher may describe tablets for the children to find; she may show tablets for the children to describe, as "I think

that tablet is 3 inches long and 2 inches wide," or the surfaces of rectangles on the blackboard may be described. In the last two instances the correctness of the judgment should be tested.

A Study of Special Surfaces. — Surfaces may be studied from two standpoints, — dimension and content. In each we should cover the three kinds of work, — imaging, comparing, and constructing.

From the Standpoint of Dimension.

Imaging.

The general lesson has taught the children to see two things about a surface, and to describe it, naming length and width. That they may get clearer images of these surfaces, we may now study them in an orderly series, starting first with the series based on $1'' \times 1''$, and then that based on $2'' \times 1''$, etc. These series will be:

$1'' \times 1''$, $2'' \times 1''$, $3'' \times 1''$ . . . . . $12'' \times 1''$

$1'' \times 2''$, $2'' \times 2''$, $3'' \times 2''$ . . . . . $6'' \times 2''$, etc.

Each series may occupy the time of one lesson, opportunity being given for arranging in orderly succession, for quick recognition, for description as to dimensions, etc. In describing a tablet, require full statements from the children; for example, "This tablet is 4 inches long and 2 inches wide." When they are more familiar with the tablets, the phrase "4 inches by 2 inches" may be used.

Comparing.

In surface work the comparison should, for the most part, be limited to equivalents and multiples, and as it depends largely upon the work in content, it will be described later.

Constructing.

In the work in surface measure the possibilities for building are much greater because of the second dimension. A few

typical exercises are given. In these the children may work at their desks, each one being provided with a box of tablets.

Exercise I.

Find a tablet 3 inches long and 1 inch wide; find one 4 inches long and 1 inch wide. Build. Ella tells what she has done in this way, "I put a tablet 3 inches long and 1 inch wide with one 4 inches long and 1 inch wide, and it made a tablet 7 inches long and 1 inch wide."

Exercise II.

Think of a tablet 4 inches long and 2 inches wide. Build, using two tablets. May says, "I put a tablet 3 inches long and 2 inches wide with one 2 inches long and 1 inch wide, and I have a tablet 4 inches long and 2 inches wide." John says, "I put two tablets each 4 inches long and 1 inch wide together, to make a tablet 4 inches long and 2 inches wide."

Exercise III.

Place a 3-inch square. Look at it. Think of a tablet 4 inches long and 3 inches wide. Build it, putting one tablet with the one you have. Describe.

These three exercises are suggestive of many which may be given. Have a definite plan in mind for the work of each day. Occasionally use more than two tablets in building. Encourage original work. After each description let the other children indicate in some way if theirs is like the one described.

From the Standpoint of Content.

Imaging.

Thus far the children have thought of a surface as having length and width. Attention must now be directed to the number of square inches it contains. The basis for this is a knowledge of a square inch, which they already have from the preceding work.

For the first lesson in content the children are gathered around the table upon which are placed the various tablets already studied and an extra supply of tablets, 1 inch square. Each child chooses a tablet and notices its length and width. The teacher directs each to find how many square inch tablets can be placed upon the one he has. These are placed with care and the results given. Helen says, "I have a tablet 4 inches long and 3 inches wide, and I can place 12 square inch tablets upon it." Ray says, "This tablet is 8 inches long and 1 inch wide, and I put 8 square inch tablets upon it." After each child has told what he has done, he chooses a different tablet and covers it with square inch tablets as before. This time as the report is given the attention is directed to the arrangement in rows. For example, Helen, who has covered a surface $4'' \times 2''$, and has reported that 8 square inch tablets are necessary, is asked to count them by four, or by two. May who has covered a tablet $4'' \times 3''$ is asked to show the threes in it. Each child is given abundant practice in covering surfaces of tablets or solids with square inch tablets until he begins to see the square inches in each surface. The new word "contains" is suggested and used by the teacher, and the child describes the tablet he has covered thus, "The tablet is 4 inches long and 3 inches wide and the surface (showing it) contains 12 square inches."

Again different tablets are chosen and each child builds with square inch tablets, one like his own, placing it a little distance from the first, not on top of it as before. Robert says, "I built a tablet 3 inches long and 2 inches wide and used 6 square inch tablets." Abbie says, "This surface, 7 inches long and 1 inch wide, contains 7 square inches." As the children describe the one built, they are asked what they think is true of the number of square inches in the tablet chosen, and the description of it is given. If a child does not see the number

readily, let him find out by placing the tablet upon the one built, or by again covering the surface with square inch tablets.

In imaging the square contents of any surface, we think of the number of square units in each row and the number of such rows. The general lesson just described has suggested that the children should be helped to see the orderly arrangement in rows. In order to still further aid them in this direction, the surfaces should now be studied again in orderly series, this time from the standpoint of content. The first series, $1'' \times 1''$ to $12'' \times 1''$, furnishes the basis for much of the work in content and should become familiar through covering surfaces and building until the children readily give length, width, and content of the surface of each. Then the series based on each of these fundamental forms may be taken in turn and the surfaces seen as a number of rows of square inches, and the contents counted by twos, threes, fours, etc., instead of by ones. After this work the children should be able to image the square inches in a surface instead of being obliged to actually place the square inch tablets.

Comparing.

Equivalents.

It is possible that in the building or in the covering of surfaces already described in connection with the work in imaging, some children will have noticed that different surfaces contain the same number of square inches; that a tablet $3'' \times 2''$ or $6'' \times 1''$ contains 6 square inches. Give them an opportunity to further compare equivalent surfaces by allowing them to build, in as many ways as they can, tablets containing a given number of square inches, to describe the different ones, and to find and describe different tablets alike in content.

Ratio.

When the children are fairly familiar with the content of surfaces, they are ready for another phase of the comparing, namely, ratio. Three steps in this work will be described: finding a *one* for any *one*, building a *two* (*three*, etc.) for any *one*, finding a *two* (*three*, etc.) for any *one*.

Finding a one *for any* one.

Upon the table around which the children are gathered are the various tablets already studied. The teacher shows one (other than the 1-inch square) and asks each child to find one as large as hers. Each does so and Robert reports, "I have found a tablet as large as yours." Showing the one she has, the teacher says, "I am going to name this tablet. I will call it a *one*. If yours is as large as this, what will you call yours?" Robert replies, "I will call this a *one*." The teacher says, "I have a *one* for your *one*, Robert, what have you for my *one?*" Robert answers, "I have a *one* for your *one*." The teacher says to different children, "I have a *one* for your *one*," and waits for a reply from each, that all may use and get acquainted with the new phrase, "I have a *one* for your *one*." Then the teacher shows a different tablet, asking the children to find a *one* for her *one*. After several children have done so and used the new phrase, "I have a *one* for your *one*," the teacher asks Ray if he has a *one* for any one's else. He looks around the table and replies, "I have a *one* for May's *one*." May is asked if she sees any one else who has a *one* for her *one*, and she sees that Robert has, and Fred has. Different tablets are shown and practice given in finding a *one* for any *one*. Each child chooses any tablet he will and, looking around the table, names those boys and girls who may have a *one* for his *one*. It is probable in this first step that those chosen will be like the one shown, in dimensions as well as in content.

Thus the *one* for a tablet $3'' \times 2''$ will be a tablet $3'' \times 2''$. If a child, seeing the content chiefly, shows a $6'' \times 1''$ tablet as the *one* for a $3'' \times 2''$, accept it but do not expect many of the children to discover this until the next work is done.

Building a two (three, *etc.*) *for any* one.

The material and conditions for this step are the same as before. The teacher shows a tablet say $2'' \times 1''$ and directs each child to find a *one* for her *one* and to place it before him. This being done, each is directed to find another *one* for the teacher's *one* and to place it with the first. To the question, "What have you for my *one?*" the answer will probably be, "I have two *ones* for your *one*." Some of the children will doubtless say, "I have a *two* for your *one*." If not, the teacher herself introduces the phrase, "a *two* for a *one*." Different tablets are chosen for the *one*, and the *two* for the *one* built in each case. When the phrase, "a *two* for a *one*," is familiar, it may be used in description. John says, "I have a *two* for your *one*. Your *one* is a tablet 4 inches long and 1 inch wide. The *two* for your *one* is 4 inches long and 2 inches wide." Abbie perhaps has built hers differently from the one John has made. She says, "The *two* for your *one* is 8 inches long and 1 inch wide." Then the teacher calls attention to the number of square inches in the surface of each. John says, "Your *one* contains 4 square inches. The *two* for your *one* contains 8 square inches." Abbie finds the same thing true. After comparing their tablets with the teacher's, John and Abbie may compare with each other's. If John says, "I have a *two* for Abbie's *two*," accept it, but suggest that there is a better way to say it. Some child may say, "Abbie has a *one* for John's *one*," if not, wait until the relation is clearly seen, when the expression comes easily. Continue the work of building, making *twos*, *threes*, *fours*, etc., for various *ones*. If the

description is to be required, limit the tablets to those containing not more than 12 square inches. Individual children may go beyond this in their original work.

Finding a two (three, four, *etc.*) *for any* one.

This may be connected directly with the preceding step, by allowing the children after building a *two* or a *three* to find one already made like the one built, to name this the *two* or the *three* for the one, and to describe as before. Later the children in many instances will be able to find a *two* or a *three* for any *one* without the intermediate step of building, though they should always go back to this when in doubt. If desired, the work in ratio may be deferred until solids are studied.

Constructing.

As the exercises in building are similar to those described before, in the work of studying surfaces from the standpoint of dimension, only one illustration is given. Notice that all the directions and descriptions emphasize the content of surface.

Exercise.

Find a tablet with a surface containing 4 square inches; find one containing 2 square inches. Build. The child says, "I put a tablet containing 4 square inches with one containing 2 square inches and built one with a surface containing 6 square inches."

This statement is true for all, though some may have put a 2-inch square with a tablet $2'' \times 1''$, making a $3'' \times 2''$ tablet, while others may have put a tablet $4'' \times 1''$, with a tablet $2'' \times 1''$, making a tablet $6'' \times 1''$. The dimensions must be given when different results are noticed. Be careful in giving the directions to name only those forms with which combinations are possible, whatever is chosen; for example, it is not always possible to put a tablet containing 4 square inches with

one containing 6 square inches; one may be a $4'' \times 1''$ and one a $3'' \times 2''$, in which case no building is possible.

Building tablets of given content using two tablets will result in repeated discovery of facts in addition. Building in sequence, starting with any form and returning to the same, will give many varieties in both dimension and content. The best way for the teacher to prepare for any of these exercises is to build with the tablets and find out for herself the possibilities and opportunities.

Perimeter. — In connection with the study of surfaces comes the work in perimeter. First, give the idea of the perimeter of a surface as the distance around it, by allowing the children to walk around a large rectangle drawn upon the floor and to trace around the edges of the tablets. Let each place the inch sticks around the edge of a tablet and after counting the number used, find the length of the perimeter by placing the same sticks beside the ruler. Repeat this with different tablets and let the children state that the perimeter of a surface 3 inches long and 2 inches wide is 10 inches. Using sticks of different lengths, choosing those as long as each side, do similar work. For example: to measure the perimeter of a $4'' \times 2''$ tablet, place two 4-inch sticks and two 2-inch sticks. When these are placed beside the ruler, count as placed, four, eight, ten, twelve; the perimeter is 12 inches. Next have the children think the length without placing the sticks, counting the perimeter as the finger traces it. Let them try to count each side as a whole, but allow them to think by ones when necessary. Count by opposites at first, then encourage counting in different ways. Exercises in counting perimeters or in finding surfaces having given perimeters will give constant practice in addition.

Seat Work. — The most valuable material for seat work in surface measure for Grade I is a supply of paper marked off in square inches. This may be prepared on a hectograph.

With this many exercises are possible, such as cutting rectangles of given dimensions, given content, given perimeter, work in ratio, cutting a *one*, *two*, *three*, etc., for any *one*. Work in drawing, building, and stick laying may also be profitably done as seat work.

Solid Measure

Material. — For this work a supply of blocks of different sizes is needed. There should be plenty of inch cubes and a smaller number of solids $2'' \times 1'' \times 1''$, $3'' \times 1'' \times 1''$—$12'' \times 1'' \times 1''$; of $2'' \times 2'' \times 1''$ and $4'' \times 2'' \times 2''$, and a few 2-inch cubes. Others such as $3'' \times 2'' \times 1''$ and $3'' \times 3'' \times 1''$ are useful but not necessary, as the larger solids can be built from the smaller.

General Lesson. — The introductory lesson aims to give the children the idea of a solid as having three dimensions, — length, breadth, and thickness or height; to help them to realize that in describing a solid three things must be told, and to show them how to trace and give the three dimensions. In showing the three dimensions it will be found that the children are less apt to make mistakes if the three starting from or leading into one corner are named. The work with solids is similar to that done in linear and surface measures and will not be described in detail.

A Study of Special Solids.

Imaging.

Solids may be studied from the standpoint of dimension and of content. One may be described thus: this solid is 2 inches long, 2 inches wide, and 1 inch thick, and contains 4 cubic inches. The faces may also be noticed and described. Give abundant opportunity for the children to handle the solids.

Comparing.

The work in ratio may be begun with solids instead of surfaces, the same plan being followed. Linear measure is not desirable for introducing ratio, as it is very limited; but after its introduction, when the idea of finding a *one* for a *one*, or a *two* for a *one*, is familiar, it should be applied to lengths, surfaces, and solids. In the later work, the reverse ratio may be given; for example, "This" (showing a solid $6'' \times 1'' \times 1''$) "is the *two;* find the *one.*"

Constructing.

Many simple exercises in building should be given. There should be abundant opportunity for original work with the child's statement of what he has done. This will often suggest to the teacher fitting questions that will lead the child to still further discoveries.

Seat Work. — The seat work in solids is largely individual, different children having the privilege of building and working out problems.

COUNTING

Counting is one means of recognizing number. It may be begun the first day of school and should continue throughout the year, for a few minutes each day. Most children, whether they enter the first grade directly from home or from the kindergarten, can count a little, perhaps to ten or twenty, or even further. Counting will not only furnish the teacher a means of finding out the number equipment of each child, but also give an opportunity for further development.

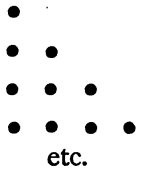
Material. — The small ball frame in common use, the larger one which may be seen across the schoolroom, and small objects of uniform shape and size may be used for this work.

Work. — The first exercises may be almost wholly imitative. The teacher shows a number of balls on the frame, and a child places the same number on the wire directly beneath; or she puts a number of blocks on the table and a child puts as many under them, etc. The actual mental processes performed by the children in doing this will vary with the stage of advancement of their number sense. The child who already knows how to count will doubtless count the number of blocks or balls placed by the teacher, and finding it to be seven, for example, will count out seven others to be placed beneath. One who cannot count will be helped by the position, placing one ball directly under each one the teacher has placed until the row beneath matches or is like the one above. This may be done without the child knowing the name seven, or even that he has seven balls in a row. In either case the child is getting practice in recognizing larger and smaller groups of numbers. These exercises in imitation may be increased in difficulty by requiring the row of objects placed by the child to be farther below the teacher's row, or at the side, or on a different part of the table; or the objects may vary, the child placing as many blocks on the table as the teacher places marks on the board.

All this work may be done without the use of number names, but it is probable that the children who are familiar with these names will use them, and the others will begin to learn from them. In order that we may be sure that each child knows the numbers in their order and associates the right name with the right number, we may now take them in their orderly succession.

Under the teacher's direction the children may arrange small objects, as blocks or colored circles: in the first row, placing one; in the second, one directly below and one to the right; in the third, two directly below those in the second

row and one to the right; and so on to ten or twelve, making this familiar form:



The number in each row should be named by a child who already knows it, and as the building continues, these names should be constantly repeated in their order by the children, each row named being pointed to. This arrangement of objects is chosen, at first, to help the child to associate the name three with the three objects making up the row, rather than with the third block in the row. When the children can point to three, eight, etc., and know that seven always comes after six, that four is always before five, etc., and begin to feel the succession by one, then the objects may be placed as commonly used, and practice given in counting various groups and different things in the room or in counting out a certain number of things asked for.

Continue the counting by ones as far as the children can, expecting that by the end of the year most of them will easily count to one hundred. Use the ball frame arranged by tens in showing the succession of numbers and learning the names, but do not expect the children to think of the numbers as made of tens and units.

When the children can count by ones easily to twenty or thirty, let them try to think by larger groups; by two, five, ten, and occasionally by the harder ones, three, four, etc. Counting by numbers other than one will be constantly used in the measure work with surfaces and solids. Much of this work may be done when the children are in a large circle around the room. One child may count the others in the ring, touching each as he

numbers; another may count the boys or the girls; the hands or fingers, etc., may be counted, or the children may arrange themselves in groups of threes or fours, thus giving practice in thinking by numbers other than one.

A little of this work in counting should be taken each day. Avoid concert work almost entirely, but let individuals have opportunity to count by the different groups, and allow each to go on as far as he can easily and intelligently. Let him work with the smaller or easier groups until the power develops to use the harder ones. Do not force any child beyond the point where his work is intelligent, for then it will become mechanical. Avoid this by keeping almost wholly to concrete work in the first grade. At the end of the first year most of the children should be able to count to one hundred by ones, tens, fives, and possibly twos, and to count by the other numbers with varying degrees of proficiency as far as each child's development allows.

Expression, or the Use of Figures

The teaching of figures in the first grade should be apparently incidental work. The child will see them on his ruler and unconsciously begin to use them. He should be taught to make them as he is taught to form the letters, and may use them in his seat work, indicating the length of the line he has drawn by the appropriate figure, numbering the squares in the rectangle he has cut. But do not require him in the first year to express the operation in figures or to do any abstract work with them.

General Suggestions for the Work of the First Grade

The language side of the work is very important. Be willing to take time for it. The training which the child gets from telling in his own words just what he has done, or has tried to do, or is going to do, is invaluable.

Be willing to advance slowly and in accordance with each child's development. Do not be anxious to find out if the child can tell you how many five and four are, but ask yourself rather if he is learning to observe accurately, if he is getting clear images, — the tools for later work, — and if he is thinking numbers intelligently and sensibly as he finds a need for them.

Let the counting and measure work go hand in hand throughout the year: a little of each every day, but generally in different exercises.

Do not keep too long on linear measure before going on to surface measures; nor too long on surfaces before studying solids. Linear measure will be constantly reviewed in the study of surfaces, and hence surfaces may be studied when the lengths from perhaps 1 to 6 inches are known. Later we may go back and study the longer measures. The same applies to work with solids.

Remember that the lessons outlined in the manual are suggestive rather than mandatory. Let each teacher put her own individuality into the work, guided by the truth that she is aiming to develop the number sense and to help the child to discover numerical relations for himself through simple constructive work with measures. She will find that doing the work with the children and studying the work of the children are her best guides.

GRADE II

The work of this grade includes a more intimate acquaintance with the numbers 1 to 100 through counting, a better understanding of the figures through their use in expressing number relations, and a broader knowledge of the operations through the constructive work with measures. It is not intended that in the schoolroom the work of either one should be completed before the next is begun. There should be some work in each every day. Each will be treated separately here, however, beginning with the measure work.

MEASURE WORK

Linear Measure

Material. — Measures 1 to 12 inches long, foot-rule, yardstick.

Review. — The first lessons should be spent in reviewing the measures 1 to 12 inches and 1 foot long, through exercises in imaging, comparing, and constructing.

Study of Lengths 12 to 20 Inches Long.

Imaging.

The length of a line between 12 to 20 inches long is determined by its relation to 12 inches. In judging such a line, we estimate 12 inches and the number of inches more. This gives us a suggestion for presenting these lengths to the children and shows us the need of careful review of the foot-rule, of the 12-inch length, and of the shorter measures.

Much of the work of the second grade may be done with the entire class at their seats, with material at each desk or on the table, as the lesson requires. For the first work have on the board a number of lines from 13 to 20 inches long, with some strings and strips of paper of the same lengths. Let the chil-

dren notice the lengths and show them how to use the foot-rule in measuring them, placing a mark carefully just above the end of the ruler, then slipping the ruler along and counting on the additional inches by ones or twos, etc. Give practice in measuring other things in the room an exact number of inches long, and in drawing lines of definite lengths. Give opportunity for judging lengths, being careful that the statement made uses the words, "I *think* this line is 18 inches long," and that each judgment is tested.

Comparing.

In the work in imaging we may introduce the different lengths 12 to 20 inches in one lesson. In comparing it is well to take each new length in order, comparing it with each of the shorter measures already known.

The study of the 14-inch measure will be taken as an illustration. Lines 14 inches long are on the blackboard, and each child has on his desk two of the 12″, 7″, 2″ measures, and one of each of the others. Attention is called to one of the 14-inch lines, its length is estimated, measured, and named. Each child is directed to build a 14-inch measure using a 12-inch measure and one other. Abbie reports, "I have built a 14-inch measure with a 12-inch measure and a 2-inch measure." They are told to keep the measure made and to think of it as one measure. Each places a 12-inch measure beside the 14-inch, and different children give the facts noticed. Similar work is done with each of the other measures. With the 7-inch and the 2-inch measures expect the multiplication and division facts to be stated as, "Two 7-inch measures make a 14-inch measure," "A 7-inch measure is one-half as long as a 14-inch measure," etc. Give an extra supply of 2-inch measures to the child who says, "I think seven 2-inch measures will make a 14-inch measure," that he may test it, and allow a row of

children to work together at one desk to prove this fact for themselves. The ratio should be seen here also. "The 14-inch measure is a *two* for a 7-inch measure;" or, when the seven 2-inch measures have been placed beside the 14-inch measure, the teacher may say, "Call the 14-inch measure a *seven*, show the *one*."

When all the relations have been seen (including that between a 13-inch measure and a 14-inch measure), exercises similar to the following may be given. Each child chooses a measure shorter than 14 inches and finds what must be put with it to make one 14 inches long. He hides the measure chosen, saying, for example, "You must put a 10-inch measure with the one I have to make a 14-inch measure." The others study their 14-inch measures and their rulers and one answers, "I think you have a 4-inch measure." The first child shows it, saying, "You are right." This exercise may be varied by having the number of inches that one measure is longer or shorter than the other, stated.

Constructing.

This part of the work should always be thought of as problem work in which there is something definite for the child to do. Thus far these definite directions have been given him orally by the teacher, and in some of the work she will still continue so to give them. But most of the problems should now be written instead of oral, that the child may become more self-reliant, able to interpret written language, fulfil conditions, and reach conclusions for himself. These problems should be expressed very simply. Experience has shown the following characteristics to be desirable.

The problems should be in the direct form: do not say, "If I put a 3-inch measure with a 4-inch measure, what shall I have?"

Avoid the third person, not: "John placed a 3-inch measure with a 4-inch measure. What did he make?"

Give a simple direction with no question in it. "Put a 4-inch measure with a 3-inch measure." After the child's work has been done and reported, the teacher may ask any question that seems helpful. Often none will be necessary.

Written problems may be introduced as early in the second grade as they can be read by the children, often in the review of the measures 1 to 12 inches long. As the work progresses, sets of problems should be prepared in the different measures from the standpoint of the operation, of ratio, and of the study of a special number. In each set there should be a different problem for each pupil.

The work may be conducted in this way. The material needed should be conveniently placed and arranged so that the children may get it readily. The problems are distributed and time is given for each child to read his problem and to think what he is to do and what material he needs. Train the child not to begin work until he knows what he is going to do. Those who have drawing to do, find their places at the board or work on the floor if it be suitable; those who know the material needed, go, ruler in hand, to the table, and the room becomes a busy workshop, each child working independently. If one hesitates, the teacher may help him with the reading of the problem if that be the difficulty; but she would better leave the working out to himself, even if he does not solve it that day. When the children find the material needed, they return to their seats, do what the problem directs, and are ready to report. Harold reads his problem, "Build a 14-inch measure with two measures not alike." He says, pointing as he talks, "I built a 14-inch measure with a 9-inch measure and a 5-inch measure." Helen reads, "Draw a line 14 inches long. Rub out 6 inches." She says, "I drew a line 14 inches long and rubbed out 6 inches,

and it left a line 8 inches long." John reads: "Find a 2-inch measure. Call it a *one*. Build a *seven* for it." He says, "Here is a 2-inch measure. This is the *one*. I have built a *seven*. It is 14 inches long, and is the *seven* for this *one*." Thus the work goes on until all have reported, or until the teacher, passing quickly around, has inspected the work of each. Those children who have solved their problems correctly write their names on the back of the ones solved, that in the next distribution they may not receive the same. A set of problems may be used for a number of lessons, until the children become familiar with them, when a new set should be taken.

Study of Foot and Yard. — The foot and yard should be familiar units in the second grade. They must be studied as the inch has been, although not in such detail, and should be used in problem work.

Surface Measure

Material. — An abundant supply of square inch tablets, other tablets containing not more than 50 square inches, and a square foot.

Review. — The first lesson should be spent in reviewing the tablets already studied, describing them, giving dimensions, content, and perimeter. Much of this review may be in the form of written problems, giving the exercises in construction used in the first grade.

Study of Larger Surfaces. — For the general class work surfaces containing not more than 50 square inches are studied, although individuals may go beyond this limit in their building. The surfaces containing 13 to 20 square inches should be studied in their order. Beyond that the work should be chiefly with the more important composite numbers: surfaces containing 21, 24, 25, 27, or 28 square inches, etc., rather than those containing 23, 26, or 29 square inches, etc.

Imaging.

In addition to describing surfaces, giving dimensions, content, and perimeter, the new and important work is to develop the power to see the larger surface as made up of smaller ones, through an exercise that may be called "wholing and parting." To illustrate this we will use a surface containing 13 square inches. Each child is provided at his seat with a number of square inch tablets from which he counts out thirteen. He is directed to build a tablet with these. Fred reports, "I have built a tablet 13 inches long and 1 inch wide, containing 13 square inches." The teacher says: "I see two tablets in yours, one is 2 inches long and 1 inch wide. Show it to me." Each child moves the one described a short distance and Harold says, "The other is 11 inches long and 1 inch wide." Helen is asked what she sees in her tablet and replies, "I see two tablets, one is 11 inches long and 1 inch wide; the other is 2 inches long and 1 inch wide." The teacher says: "Put the tablet containing 2 square inches with the one containing 11 square inches." A child reports what he has done and the result. Similar work is done with any two tablets making the $13'' \times 1''$, the teacher directing the work at first, later the children describing and showing what they see. Sometimes each child may simply show, by separating, any two tablets, and the teacher may describe what she sees, the child indicating by rising, if the tablets described are on his desk.

With the composite numbers there will be greater possibilities in wholing and parting. For instance, when 16 square inch tablets are used in building, the resulting form will vary. Some will have the $16'' \times 1''$, some an $8'' \times 2''$, some a 4-inch square. After these have been described all may build the 4-inch square and the teacher may suggest that she sees four equal tablets. When shown, one child may have four tablets each $4'' \times 1''$, another four 2-inch squares. Each contains 4 square inches

and when put together and described from the standpoint of content, the same truth is emphasized. In the work of wholing and parting with the 4-inch square, these facts will be illustrated:

$12 + 4$, $8 + 8$, $16 - 12$, $16 - 8$, 4×4, 2×8, $16 \div 4$, $16 \div 8$;

with the $8'' \times 2''$:

$14 + 2$, $12 + 4$, $10 + 6$, $8 + 8$, $16 - 2$, $16 - 4$, $16 - 6$, $16 - 8$, 8×2, $16 \div 2$;

with the $16'' \times 1''$, in addition to those already given:

$15 + 1$, $13 + 3$, $11 + 5$, $9 + 7$, $16 - 1$, $16 - 3$, $16 - 5$, $16 - 7$, 16×1, $16 \div 1$.

These facts are not to be memorized at this stage apart from the measure work, but let there be abundant opportunity for repeated discoveries of these truths through parting and wholing. The continuation of the work with the composite numbers to fifty will afford much practice in the operations. Counting perimeters will give many combinations in addition.

As this work progresses the children should become very quick in handling material and in seeing larger surfaces as made up of smaller ones. Some will doubtless see the possibility of showing a fact more clearly by an additional movement of the tablets, as "In a 4-inch square, I see a 3-inch square containing 9 square inches and with what is left I can make a tablet 7 inches long and 1 inch wide, containing 7 square inches."

Comparing.

Through Addition and Subtraction.

In this work the children may begin to compare the dimensions and contents of tablets between which no simple ratio exists. For example: Harold may have a tablet $5'' \times 2''$,

May one 3″ × 2″. Harold may say, "This tablet is just as wide as May's, but it is 2 inches longer." May compares content, saying: "The surface of Harold's tablet contains 10 square inches, mine contains 6 square inches. Harold's contains 4 more square inches than mine." In selecting surfaces for this work, choose those in which the differences may be easily seen.

Through Ratio.

When the ratio is taken up in the second grade in connection with surface work, the first exercise may profitably be conducted around the number table, when the familiar tablets are reviewed from the standpoint of ratio to see if the children recall the idea readily. Let them have practice in building the larger tablets, as a *six*, *seven*, etc., for a *one*. The ratio work should be carried on systematically so that all the multiplication facts may be covered. The table of threes is chosen as an illustration. The teacher shows a tablet 3″ × 1″ and says, "This is a *one:* Fred may find a *two* for my *one;* Abbie a *three;* May a *three* different from Abbie's; Harold a *four*," etc. Have the children understand that they may always build the desired measure if they cannot find it. When ready, the report is given somewhat in this way. Fred says: "I have a *two* for your *one*. Your *one* is a tablet 3″ × 1″, containing 3 square inches. My *two* is a tablet 6″ × 1″, containing 6 square inches." Abbie says: "I have a *three* for your *one*. Your *one* contains 3 square inches; my *three* contains 9 square inches," etc. Then the teacher may put different questions to those whose work will reveal the answer. To Fred, "The *one* contains 3 square inches, the *two* must contain ——." To Abbie, "The *three* for a tablet 3″ × 1″ may be ——." Abbie, who has a 9″ × 1″, answers, "It may be a tablet 9 inches long and 1 inch wide." "Or, it may be ——." May, who has a 3-inch square, answers, "It may be a 3-inch square." "But a *three* for a tablet contain-

ing 3 square inches must contain ——." Both Abbie and May have the answer, "A *three* for a tablet containing 3 square inches must contain 9 square inches."

In the ratio work in surfaces, require the class as a whole to work only as far as 50 square inches, but expect many individuals to go beyond this in their original problems.

Constructing.

What has been said of written problems in connection with linear measure applies equally to the work in surface. Sets of problems should be prepared and suggestive ones will be given later. Original problems should be encouraged.

Study of Foot and Yard. — A little work with the square foot and square yard may be done in this grade.

Solid Measure

The work in solids follows the same plan as that in surface measure, and is so similar that only general suggestions will be given. The details may best be worked out by each teacher with the solids in her hands.

Material. — An abundant supply of inch cubes, a box of solids.

Review. — Description of solids already known.

Study of Larger Solids.

Imaging.

Description of larger solids as to dimension and content. Work in wholing and parting.

Practice in seeing solids as made up of layers of cubic inches and in counting contents thus.

Description of faces and counting of entire surface; for example, a solid $4'' \times 2'' \times 2''$. "This solid is 4 inches long, 2 inches wide, 2 inches thick, and contains 16 cubic

inches. It has 6 faces" (pointing to each as described), "four are 4 inches long and 2 inches wide, each containing 8 square inches, two are 2-inch squares, each containing 4 square inches. The entire surface contains 8, 16, 24, 32, 36, 40" (touching each face as it is counted), "40 square inches."

Comparing.

Equivalents.

Building and description of solids alike in content but different in dimensions.

Ratio.

Work similar to that done in surface measure, covering the multiplication facts again.

Constructing.

Class Work. — Building solids before the class, different children describing the starting form, the solid to be put with this to make a larger one of given dimensions, finding and placing the required solid, and describing the resulting form. Improve the opportunity for coöperative work in construction.

Individual Work. — Sets of problems as in the other measures.

Expression of Operations by Means of Figures

In the second grade the expression of addition, subtraction, multiplication, and division is begun. The columnar form is the one taught, as that is more commonly used in school work and in later life. It is taken wholly in connection with the solution of problems, and aims to be an intelligent expression of work actually performed by the child.

An exercise in addition will illustrate the work. Each child is supplied with ruler, chalk, and a piece of cloth to serve as an eraser. These should be always at hand. Each has also a set

of linear measures 1 to 12 inches long. The class is already familiar with figures and can solve written problems, but has not expressed the operations in figures. The teacher writes a problem on the board, waiting after each sentence for the children to do as directed. Each child uses the floor by his desk as a blackboard. Suppose this to be the problem: "Draw a line 7 inches long. Draw another 4 inches long. Draw one as long as both together." When all are ready, John reads the problem and says, pointing as he talks, "I drew a line 7 inches long, I drew another 4 inches long, and I drew another as long as both together, and that is 11 inches long." The teacher asks for the length of the first line drawn, and as it is given writes the figure 7 on the board, saying, "This stands for the length of that line." She does the same with the next line, placing the figure 4 below the 7 with the plus sign to stand for putting both together. The short line is placed beneath, and as a child gives the length of the line he has drawn which is as long as both together, 11 is written. Each child expresses his work in the same way. Then the teacher reads the problem, explains as John did, and adds, pointing to figure and line as she talks, "7 stands for the length of this line; 4 stands for the length of this line; 11 stands for the length of this line as long as both together; 7 and 4 are 11." Several children do the same. Other problems are given in drawing lines or in building measures, the expression in figures being written by each child after he has solved his problem. Different children tell what each figure stands for. When they have become familiar with the form of the expression through class work, a set of problems in addition is given, choosing those which make the putting together emphatic. Each one expresses his own problem when solved.

Similar work in subtraction, multiplication, and division is done, care being taken in each to choose first those examples which illustrate the process clearly.

Typical problems in each are given.

Addition. — "Find a tablet 3 inches long and 2 inches wide. Count the perimeter." The work is expressed thus:

$$\begin{array}{r} 3 \\ 3 \\ 2 \\ +2 \\ \hline 10 \end{array}$$

and the child reads the problem and says, "I found a tablet 3 inches long and 2 inches wide, and the perimeter is 10 inches long. 3 stands for the length of this side" (pointing as he talks), "3 for the length of this side, 2 for the length of this side, 2 for the length of this side: 10 stands for the perimeter." He counts the perimeter from the tablet or from the figures expressed — "3, 6, 8, 10," or "2, 4, 7, 10."

Subtraction. — "Draw a line 12 inches long. Mark off 9 inches." The work is expressed thus:

$$\begin{array}{r} 12 \\ -9 \\ \hline 3 \end{array}$$

and explained in this way: "12 stands for the length of this line, 9 stands for the number of inches marked off, 3 stands for the length of the line left. 9 from 12 leaves 3."

Multiplication. — "Put five 3-inch measures together."

$$\begin{array}{r} 3 \\ \times 5 \\ \hline 15 \end{array}$$

"3 stands for the length of each measure, 5 tells how many measures I have, 15 stands for the length of all together. 5 times 3 are 15."

Division. — "Draw a line 16 inches long. Mark off the 2-inch lines in it." $2\overline{)16.}$ "16 stands for the length of this line, 8

2 shows the length of each line marked off, 8 tells how many 2-inch lines there are. There are eight 2's in 16."

Illustrations of the expression for ratio and for the contents of surfaces and solids are also given.

"A rectangle containing 8 square inches is the *one*. Find the *three*." The work is thus expressed: 8. 8 shows the number of square inches in the *one*, 3 tells the number of such *ones* in the *three*, 24 shows the number of square inches in the *three*.

$$\begin{array}{r} 8 \\ \times 3 \\ \hline 24 \end{array}$$

"A rectangle containing 28 square inches is the *four*. Find the *one*." $4\overline{)28.}$ 28 shows the number of square inches in the 7

four, 4 tells how many *ones* there are, 7 shows the number of square inches in each *one*.

The area of a surface $6'' \times 4''$ is thus expressed:

$$\begin{array}{r} 6 \\ \times 4 \\ \hline 24 \end{array}$$

6 stands for the number of square inches in each row, 4 stands for the number of such rows, 24 stands for the number of square inches in the entire surface.

The expression for the content of a solid 4 inches long, 3 inches wide, and 2 inches thick is:

$$\begin{array}{r} 4 \text{ cubic inches} \\ \times 3 \\ \hline 12 \text{ cubic inches} \\ \times 2 \\ \hline 24 \text{ cubic inches} \end{array}$$

4 stands for the number of cubic inches in one row, 3 stands for the number of such rows in a layer, 12 stands for the number of cubic inches in each layer, 2 stands for the number of such layers, 24 stands for the number of cubic inches in the solid. The appropriate words, "cubic inches" or "square inches," may be written if desired.

After the expression has been taught it should be used as the final step in all problem work. The figure work is really the third form of expression, showing that the child is interpreting the problem aright: first he expresses his thought with the material, next with words, telling what he has done, and lastly with figures. *The last is useless at this stage unless it stands for some operation actually performed.*

In connection with the work in original problems, the expression may sometimes be written on the board, and the children may give problems suggested by the expression.

COUNTING, OR RECOGNITION OF NUMBERS

In the second grade more time is allowed for this part of the work, perhaps twelve minutes each day, or a little less than one-third of the time given to the number work. As the counting progresses many of the children will think without objects and this will lead to the abstract work.

Extension of the Work of Grade I. — The first work is a continuation of that done in the preceding grade, extending the counting by any number, 1 to 12, to 100, starting with the number itself or a multiple. This should be done gradually, in accordance with the child's development, being careful to force no child to use a larger number in thinking on by groups than he can intelligently. Memory work has a place here, and rightfully, but let it be memory based on thought. That it may not become mere verbal memory, vary the group by which the counting is being done.

For example: suppose the class counts readily by ones, twos, fives, and tens. John is asked to count by tens to thirty. He says, " 10, 20, 30." Abby is told to think on by 5 to 50. She says, " 35, 40, 45, 50." Harold goes on by 1 to 55; Fred by 5 to 70; May by 2 to 86; Helen by 1 to 90; and Robert by 10 to 100. All must be alert and may have a part, the teacher fitting the work to the ability of each child. Much practice may thus be given in thinking by different groups. As each new number is taken it may be used first with the lower and easier numbers, then with the harder combinations. This work is closely connected with the work in surfaces in counting perimeter, and with the work in solids in counting faces or the area of the entire surface.

If the children are able, some of the work in counting may be done, starting with any number; as beginning with 1, thinking on by 2, or starting with 25, thinking on by 3. But if this is found to be difficult, go back to simpler work and develop more power, leaving this to be done later.

Counting backward may be taken to a limited extent.

Counting by Ten as a Unit and by Tens and Ones. — Thus far we have paid little attention to the arrangement of numbers in a decimal system: we have counted by 10 as by 5 or 3, except that 10 is found an easier number to count by than most of the others. In the second grade we study 10 as a unit and become acquainted with numbers as made up of tens or of tens and units.

Material. — Splints or short sticks fastened in bundles of tens, each pupil having 10 bundles.

Study of Numbers composed of Tens. — Each child counts the number of sticks in several different bundles, is told that the bundles are all alike, and finds that the easiest way to show 10 sticks is to pick up a bundle or a 10. They count the tens; 1 ten, 2 tens, 3 tens, etc.; or show 5 tens, 9 tens, etc. They

count by tens, " 10, 20," etc., and notice the significance of the name, 60 or 6 tens, 70 or 7 tens, 20 or 2 tens, etc. The number is expressed in figures and the significance of the figure and its position emphasized; 2 tens — 20, 9 tens — 90.

Numbers composed of Tens and Units. — Similar work is done with tens and ones until the children recognize 3 tens and 5 as 35; think of 87 as 8 tens and 7; express in figures the number of tens and units shown, realizing something of the significance of the position of each figure; find the correct number of objects when figures are written; read the expression in two ways, as: 47 — forty-seven, or four tens and sev[illegible].

Expression of Numbers by the Use of Figures

The work of expression should be taken in connection with the recognition of numbers as suggested above and should emphasize the significance of the position of the figures.

Seat Work

Exercises similar to those suggested for use in the first grade, but adapted to the power of the children will be found useful. Descriptions of tablets or of solids may be written. Rectangles of given dimensions or content may be drawn. Exercises in ratio may be worked out with square inch tablets and the different results given in writing. Problems like the following may be distributed: a 5-inch measure and a — inch measure make an 11-inch measure. The children, using foot-rules or measures, solve the problems, then write the statement, filling in the blanks with the correct figures. Counting by any number, as 4, 5, etc., may be expressed in figures instead of orally. This should be the only abstract figure work done in the second grade.

General Suggestions for the Work of the Second Grade

The importance of the language side of the work is as great as in the first grade. Require complete statements of work done.

Visualizing is the basis for successful work in problems. Thorough work in imaging and comparing is the best preparation for constructing.

Train the child to image conditions and to think of the material needed before beginning work in problems.

Do not ask questions to help a child to solve his problem. Independent work is more valuable even if at the end the child says, "I am wrong."

Give little or no abstract drill in number facts in this grade. The time is better spent in the repetition of activities rather than in the repetition of verbal drills.

GRADE III

MEASURE WORK

Material. — For linear measure, a foot-rule for each pupil and several yardsticks.

For surface measure, series of tablets:

$1'' \times 1''$ to $12'' \times 1''$
$1'' \times 2''$ to $12'' \times 2''$
$1'' \times 3''$ to $12'' \times 3''$
etc., to
$1'' \times 12''$ to $12'' \times 12''$

For solid measure, a supply of cubic inches; twelve each of $2'' \times 1'' \times 1''$ to $12'' \times 1'' \times 1''$, and a few $2'' \times 2'' \times 1''$, $2'' \times 2'' \times 2''$, $2'' \times 2'' \times 4''$.

Review. — Review lengths, surfaces, and solids already studied through description and written problems.

Linear Measure.

Foot and Yard.

Study the foot and yard, using these units in measuring longer distances in the schoolroom, in the corridors, and out of doors. Use either or both units in problems.

Drawing to a Scale.

Introduce the drawing of lines and plans to a scale. For example: suppose the children have measured a garden plot in the school yard and found it to be 9 feet long and 4 feet wide. Let the plan of this plot be drawn in the schoolroom, first of actual size. In order that the class may learn to interpret the lines of a plan, let them point to the line which stands for the south side of the garden, walk along the line which shows the length of the garden, etc. Provide each pupil with paper

and suggest a reason for the necessity of drawing the plan on paper instead of on the floor. Let the pupils find out for themselves the difficulty in the way and its solution. Have each child draw the plan, using the scale decided upon by the class. Ask questions to make sure that the children are connecting the lines in the plan with the corresponding lines in the real garden. Draw the plans of different surfaces, using simple scales, as 1 inch for 1 foot, 1 inch for 2 feet, etc.

Surface Measures.

Study of Surfaces $1'' \times 1''$ to $12'' \times 12''$.

Imaging.

Give practice in describing surfaces, having the pupils state dimensions, area, and perimeter. Name the area and have the children give the dimensions; or give the area and one dimension, letting the children name the other dimension.

As the work goes on this may be done without objects. The teacher writes on the board the number of square inches in a given surface and the children name the different possibilities. Many questions, requiring constant thought and alertness on the part of the class, may be asked by the teacher during this exercise. To illustrate: "36 square inches" may be written on the blackboard. One child says, "It may be a surface 9 inches long by 4 inches wide;" another says quickly, "Or it may be a 6-inch square;" and so on until all the possibilities are given. The teacher puts these questions rapidly: "A surface $9'' \times 4''$ must contain —," "A surface containing 36 square inches, that is 3 inches wide, must be —," "A surface 36 inches long containing 36 square inches must be —," "A surface containing 36 square inches may be —," "Or it may be —." "But if it is a square, it must be —," etc. Treat other composite numbers to 100 in the same way, especially those having many factors.

Comparing.

Through Addition and Subtraction.

This work may be done with those surfaces between which no simple ratio exists, as — compare a surface $7'' \times 6''$ (A), with one $9'' \times 4''$ (B). The length of A is 2 inches shorter than the length of B. The width of A is 2 inches longer than the width of B. The perimeter of A is just as long as the perimeter of B. A contains 42 square inches, B contains 36 square inches. A contains 6 more square inches than B.

Through Ratio.

The work in ratio gives an opportunity to cover all the facts in the multiplication and division tables, and should be carried on as was the work in the second grade.

Constructing.

The work with written problems is a very important part of the measure work of the third grade. Care should be taken in the expression of the operations and a variety of problems requiring the same operation, given.

Study of Square Foot and Square Yard.

The area of larger surfaces measured in feet or yards should be found, the difference being merely in the unit of measure used, not in the method of measuring.

In this grade let the pupils cut or draw different rectangles containing 1 square foot, 2 square feet, etc.; also rectangles 1 foot square, 2 feet square, etc.

Solid Measure.— The work in solid measure follows the same general plan. The building and description of solids, the counting of the area of entire surfaces, and the ratio work will afford abundant practice in the operations. Sets of problems should be prepared covering the various kinds of work in the different measures. Each set should make use of one of the numbers

from 2 to 12 as a number measure, so that all of the fundamental facts in addition, subtraction, multiplication, and division may be covered.

Recognition and Expression of Numbers

The new unit in the third grade is the unit hundred. This is presented as a bundle of sticks made by binding together 10 tens. The numbers composed of hundreds, those composed of hundreds in combination with smaller numbers, and the expression of these numbers by figures should be studied. The work is similar to that done in studying the tens in the second grade.

Abstract Work

Through the first two grades, the work in the four operations is almost wholly concrete and the greater part is done in connection with the measure work. But the fundamental facts in addition, subtraction, multiplication, and division must be known and known thoroughly apart from measures. In order that a pupil may do satisfactory work in the grammar grades, he must know these facts accurately and be able to recall them quickly. To do this, much practice or drill is necessary. No matter how much concrete work is done, nor how many repeated discoveries of fundamental number facts are made in the constructive work with measures, we cannot depend upon concrete work alone to fix these facts. The constructive work should make it possible for the child to think numbers intelligently and thus give a strong foundation for the later work, but it must be supplemented by drill in abstract work. This drill cannot all be given in the primary school. The grammar school must do its share. But part of it belongs to the primary school, and it is with this part that we have to do. The plan of work outlined in this manual places this abstract drill in the third grade. It is

believed that the time of the first two grades would be better spent in the concrete measure work, with only the little abstract work that counting may introduce. Not that little children cannot memorize abstract number facts. This is not the question. But rather: Can they do this work with profit? Is this the best way for them to be employed? There may be certain children who may profitably take the abstract work earlier, but it is being felt more and more strongly that for the greater number of children it would better come later. It is placed, therefore, in the third year and is a most important part of the work of that grade. It is suggested that half the time allotted to number in this grade be given to the work with measures and half to the abstract work in the operations, that the two kinds of work be taken separately, and at different times, and that no objects be used during the period of abstract work.

Counting. — In this grade the pupils should be able to do the work in counting without objects. The work of the second grade should be reviewed, giving practice in counting by one number through each table: as counting by 7 to 84; and in counting by different numbers, as — by 7 to 21, continuing by 3 to 33, by 11 to 55, by 5 to 80, by 4 to 96, by 12 to 144. Introduce, also, to a greater extent than was possible or desirable in the second grade, counting by any number, starting with any number other than the number itself or a multiple.

In this work use also the different units, thus: Give to three children the same number, as 123, and ask one to count by 1, another by 10, and the third by 100.

Let children choose their own starting point and the number by which to count, other children finding out the unit chosen.

Let them count from the blackboard as the teacher writes in columns:

7 9
7 9
7 9
7 3 etc.
7 10
5 10
5 4

This will be found to give excellent practice in quick columnar addition.

Fundamental Facts in the Four Processes. — Each fact in addition carries with it the reverse addition and the two correlated facts in subtraction; for example, the fact that $7+5=12$ suggests $5+7=12$, $12-5=7$, $12-7=5$. The same is true of multiplication and division. Considering that the facts in which we add to or subtract from 10, or multiply or divide by 10, are covered in studying the make-up of the numbers, the number of facts to be learned in each operation is 45. (If the multiplication and division tables are taken through 12 instead of through 10, there will be a slight increase in the number of facts in these.) Upon these fundamental facts depends all the later work in the four processes, hence they should be accurately known. Oral drill is good in helping to fix these facts, but do not depend upon it alone. It appeals to the ear and so helps those children who learn more readily by hearing than by sight. But experiments prove that many people learn more readily through sight than hearing, and hence there should be exercises in visual drill. The power to visualize is found helpful in the later work in mathematics, so that such exercises should be given, not only to increase this power in those children who already depend upon sight rather than hearing, but even more to help those whose visualizing power is weak.

The following exercises will illustrate the work. The teacher writes upon the board:

$$\begin{array}{r} 9 \\ +\,8 \\ \hline 17 \end{array}$$

and directs the children to look at it and then close the eyes and try to think just how it looks, to try to see it as plainly as if the eyes were open. Then the eyes are opened that they may compare the real picture with the mental one. The figures

are erased and then replaced by one of the children, the others deciding whether or not he is correct. The teacher rewrites the example, with another beside it, as:

$$\begin{array}{rr} 9 & 5 \\ +8 & +6 \\ \hline 17 & 11 \end{array}$$

and similar work is done. If two can be visualized and written correctly, try three or four. After a little work, a third grade should be able to take five or six examples in one exercise. Instead of erasing all, while the children's eyes are closed occasionally erase one or two only and see if those can be rewritten in their proper places. Always work with the example as a whole, writing the answer as a necessary part each time so that the complete image

$$\begin{array}{r} 4 \\ +8 \\ \hline 12 \end{array}$$

may be mentally photographed, until from seeing the whole many times this

$$\begin{array}{r} 4 \\ +8 \\ \hline \end{array}$$

will seem incomplete and will immediately suggest 12. Do not continue the exercise beyond the point at which the close attention of the class can be held. An exercise in which concentrated attention is given is the only one of value in this work. Use the columnar form, rather than the equational, in writing the examples. For later work to follow these exercises in visualizing, cards with figures large enough to be seen across the room may be prepared, the example being written without the answer. The card should be held before the class only for an instant, the answer being immediately given by one child. If the class hesitates, go back to the exercises

in visualizing. When the answers can be given quickly, let the examples be applied to outside problems which may be given by the teacher or pupil.

Series Work. — The fundamental facts in addition and subtraction are the only ones necessary for later work in these processes. But the connection of these facts with the larger numbers must be recognized and series work will help to reveal this. Let such additions be given as:

$$\begin{array}{rrrrrr} 3 & 43, & & 5 & 25, & \\ +4 & +4 & & +3 & +3 & \\ \hline \end{array} \qquad \begin{array}{rr} 4 & 64 \\ +5 & +5 \\ \hline \end{array}$$

until the rhythm and connection begin to be felt. Then take an unbroken series, as:

$$\begin{array}{rrrrrrrrrr} 5 & 15 & 25 & 35 & 45 & 55 & 65 & 75 & 85 & 95 \\ +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 & +2 \\ \hline 7 & 17 & 27 & 37 & 47 & 57 & 67 & 77 & 87 & 97 \end{array}$$

Have a number of such series given by the class and by individuals. Do not be troubled if these are given with a rhythmic singsong. It is only the vocal expression of the underlying law. Let different series be written as well as given orally, until the children realize that knowing

$$\begin{array}{r} 3 \\ +2 \\ \hline \end{array}$$

gives the answer to

$$\begin{array}{r} 93 \\ +2. \\ \hline \end{array}$$

Take the second type, based on those additions making the ten:

$$\begin{array}{rrrr} 7 & 17 & 27 & 37 \\ +3 & +3 & +3 & +3 \\ \hline 10 & 20 & 30 & 40 \end{array} \text{ etc.}$$

and do similar work until the children feel the underlying law; namely, that whenever the sum of the units is 10, the result will be always carried to the next higher 10. Do not expect this law to be formulated, but felt and acted upon.

The third type is based on those additions whose sum is over ten:

$$\begin{array}{rrrl} 5 & 15 & 25 & \\ +9 & +9 & +9 & \text{etc.} \\ \hline 14 & 24 & 34 & \end{array}$$

and reveals the law that when the sum of the units is over 10, the result is carried just as many units over the next higher 10 as the first addition shows.

Subtraction has the three corresponding series:

$$\begin{array}{rrrrrrrrr} 7 & 17 & 27 & 37 & & & 97 & & \\ -4 & -4 & -4 & -4 & \text{etc.} & & -4 & & \\ \hline 3 & 13 & 23 & 33 & & & 93 & & \end{array}$$

$$\begin{array}{rrrrrrrrr} 10 & 20 & 30 & 40 & 50 & 60 & & 100 & \\ -2 & -2 & -2 & -2 & -2 & -2 & \text{etc.} & -2 & \\ \hline 8 & 18 & 28 & 38 & 48 & 58 & & 98 & \end{array}$$

$$\begin{array}{rrrrrrrrr} 11 & 21 & 31 & 41 & 51 & 61 & & 101 & \\ -5 & -5 & -5 & -5 & -5 & -5 & \text{etc.} & -5 & \\ \hline 6 & 16 & 26 & 36 & 46 & 56 & & 96 & \end{array}$$

After the work has been taken in series, miscellaneous examples should be given.

Written Work. — By this is meant the abstract work in the four processes with numbers below 1000:

$$\begin{array}{lll} 421 & 602 & 42 \\ 382, & -378, & \times 7, \\ \underline{159} & & \end{array} \qquad 4)\overline{568}$$

The teaching of this is so familiar that no suggestions are necessary.

Seat Work

The seat work in connection with measures may include written descriptions of tablets or solids, making of original problems, drawing plans to a scale, etc. In connection with the abstract work, exercises in counting, in series work, and in written examples may be given.

General Suggestions for the Work of the Third Grade

The time spent in arithmetic in this grade should be equally divided between measure work and abstract work. It is better to give these at different times in the programme and to use no objects during the period for abstract work.

Use visualizing in fixing the number facts.

Accuracy and speed are both desirable in abstract work, but the first should never be sacrificed to the second.

Do not neglect the language side of the work.

Continue the solution and expression of directed and original problems.

The usual work of the primary grades in mensuration of time, of value, of dry and liquid substances, should be done.

The knowledge of numbers should be applied to the solution of problems outside of measure work.

A primary arithmetic suitable for the third grade may be used with profit in connection with the work outlined.

OUTLINE OF NUMBER WORK FOR PRIMARY GRADES

GRADE I

I. Recognition of the numbers.

1. By counting.

a. Teach the pupils to count by ones to 100; to know what number comes before and after a given number; to form a number from the preceding or succeeding one.

b. Give them opportunities to use this knowledge in counting out a number of objects when the name is given and in finding how many there are in a given number of objects.

c. Teach them to count to 100 by 10 and 5; to count by 2, 3, 4, 6, etc., as far as desirable.

2. By operations, through simple constructive work with measures.

a. Linear measures.

1) Introductory exercises for general comparison.

2) Study of exact lengths.

a) Teach measures 1 inch to 12 inches long.

(1) Image.

Aim to give a clear visual image of each length through seeing it, using it in measuring, trying to draw lines as long, and testing results.

(2) Compare.

(*a*) Aim to have the child grow from the general — "this measure is longer than this" — to the exact — "a 4-inch measure is 3 inches longer than a 1-inch measure."

(Note. Do not expect all to have the same power in seeing relations. Do not force the child who sees only the general relation of longer or shorter to repeat words about an exact relation.)

(*b*) Ratio.

Find a *one* for any *one*.

Build a *two* (*three*, etc.) for any *one*.

Find a *two* (*three*, etc.) for any *one*.

(3) Construct.

Directed and original problems.

b) Teach foot, foot-rule, and its use in measuring.

b. Surface measure.

1) Introductory exercises to give the idea of surface as having two dimensions.

2) Study of special surfaces.

a) Image.

Dimensions.

Content.

Perimeter.

b) Compare.

Through dimensions.

Limited amount.

Through content.

Various surfaces equal in content though not in dimensions.

Work in ratio like that in linear measure but with greater possibilities because of second dimension.

c) Construct.

Directed and original problems.

c. Solid measure.

1) Introductory exercises to give the idea of solids as having three dimensions.

2) Study of special solids.
   a) Image.
   b) Compare.
   c) Construct.
   (Note. The work with solids is similar to that suggested for the other measures, but is broader and gives constant opportunities for using both linear and surface measures.)

II. Expression of the numbers by figures.
Taught incidentally as needed. No formal figure work during the first year.

III. Suggestions for seat work.
Many kinds of busy work are possible in connection with this measure work: cutting, drawing, sorting, building, etc. It should be constructive work with some form of material, and not abstract figure work.

GRADE II

I. Recognition of numbers 1 to 100.
1 Counting by any number (1 to 12) to 100.
   a. Starting with number itself or a multiple.
   b. Starting with any number.
2. Counting backward.
   (Limited amount.)
3. Counting by tens.
   Recognition and use of ten as a unit.
4. Counting by tens and ones.

II. Expression of numbers 1 to 100.
Notice the significance of the position of figures.

III. Operations with numbers.
Through constructive work with measures.
1. Linear measure.

a. Review measures 1 inch to 12 inches long, through imaging, comparing, and constructing.

b. Study of lengths 12 inches to 20 inches long.

1) Image.

Refer to foot length.

Use of foot-rule in drawing and measuring lines longer than 12 inches.

2) Compare.

a) Through addition and subtraction.

b) Through ratio.

3) Directed and original problems.

Problems should now be given to the pupils in writing instead of orally, and should be solved independently.

c. Study of foot and yard.

Similar work.

2. Square measure.

a. Review tablets known, giving dimensions, contents, perimeter.

b. Study of larger surfaces not exceeding 50 square inches.

1) Surfaces containing 13 to 20 square inches.

2) Surfaces containing 21, 24, 27, 28 square inches, etc., using important composite numbers only.

In studying (1) and (2)

a) Image.

"Wholing and parting," describing parts of which whole is made.

b) Compare.

Through addition and subtraction.

Through ratio.

c) Construct.

Directed and original problems.

c. Study a square foot and square yard.

3. Solid measure.
   a. Image.
      1) Solids described as to dimensions, content, and faces.
      2) Counting content of surfaces.
      3) Wholing and parting.
   b. Compare.
      1) Equivalents.
      2) Ratio.
   c. Construct.
      Directed and original problems.

IV. Expression of operations by figures.
   Taken in connection with solution of problems.
   Aim — an intelligent expression of work done.

V. Suggestions for seat work.
   Work similar to that of Grade I., but more advanced is desirable. Make the work concrete. Use abstract figure work only in connection with counting.

GRADE III

I. Recognition of numbers.
   1. Of hundreds.
   2. Of hundreds in combination with tens and units, with tens, with units.

II. Expression of the same.

III. Operations.
   1. Measure work.
      a. Review lengths, surfaces, and solids, through description and problems.
      b. Linear measure.
         1) Review of the work with inches.
         2) Foot and yard.

a) Image, compare, construct.
b) Problems using one or more units.
c) Drawing lines and plans to a scale.

c. Surface measure.
1) Study of surfaces $1'' \times 1''$ to $12'' \times 12''$.
a) Image.
(1) Description of each as to dimensions, area, perimeter.
(2) Area given, dimensions named.
(3) Area and one dimension given, other dimensions named.
Some work without objects.
b) Compare.
(1) Through addition and subtraction.
(2) Through ratio.
Variety in questioning.
Possibilities in multiplication and division.
c) Construct.
Directed and original problems.
2) Square foot and square yard.

d. Solid measure.
Similar work.

2. Abstract work.
Half the time to be given to this: no objects to be used during the period of abstract work.
a. Counting.
b. Fundamental facts in the four processes.
Use visualizing in fixing facts.
Use columnar form in expression.
Apply to outside problems.
c. Series work.
d. Written work.
Accuracy first; then speed.

Material

The amount of material given is that required for one class. For the linear measure, sticks, splints, or strips of cardboard, one-half an inch wide of the different lengths; for the surface measure, tablets cut from manila paper or cardboard; and for the solid measure, wooden blocks of the dimensions given, are necessary. These must be cut with great accuracy. A cutting machine is desirable for each school so that the teachers may prepare material as needed.

GRADE I

Linear Measure.

300	1-inch measures	25	7-inch measures
150	2-inch measures	25	8-inch measures
100	3-inch measures	25	9-inch measures
75	4-inch measures	25	10-inch measures
50	5-inch measures	25	11-inch measures
50	6-inch measures	25	12-inch measures

A ruler for each pupil.

Surface Measure.

300	tablets $1'' \times 1''$	50	tablets $2'' \times 3''$
150	tablets $2'' \times 1''$	25	tablets $2'' \times 4''$
100	tablets $3'' \times 1''$	25	tablets $2'' \times 5''$
75	tablets $4'' \times 1''$	25	tablets $2'' \times 6''$
50	tablets $5'' \times 1''$	25	tablets $3'' \times 3''$
50	tablets $6'' \times 1''$	25	tablets $4'' \times 3''$
50	tablets $2'' \times 2''$		

Solid Measure.

One box for each grade.

GRADE II

Linear Measure.

50	1-inch measures	25	7-inch measures
50	2-inch measures	25	8-inch measures
50	3-inch measures	25	9-inch measures
50	4-inch measures	25	10-inch measures
50	5-inch measures	25	11-inch measures
50	6-inch measures	25	12-inch measures

A foot-rule for each pupil.
Several yardsticks.

Surface Measure.

A supply of inch square tablets.

25 each of $2'' \times 1''$ to $12'' \times 1''$ tablets
25 each of $2'' \times 2''$ to $6'' \times 2''$ tablets
25 each of $3'' \times 3''$ to $4'' \times 3''$ tablets

A few of the larger tablets containing not more than 50 square inches.
A square foot.

Solid Measure.

A box of solids.
Supply of inch cubes.

GRADE III

Linear Measure.

Foot-rules.
Several yardsticks.
A surveyor's tape.

Surface Measure.

One of each of the following tablets:

$1'' \times 1''$ to $12'' \times 1''$
$1'' \times 2''$ to $12'' \times 2''$
$1'' \times 3''$ to $12'' \times 3''$
etc. to
$1'' \times 12''$ to $12'' \times 12''$

Solid Measure.

Ten or twelve each of $1'' \times 1'' \times 1''$ to $12'' \times 1'' \times 1''$.
Ten or twelve of $2'' \times 2'' \times 1''$.
Six each of $2'' \times 2'' \times 2''$ and $4'' \times 2'' \times 2''$.

One each of the 3-, 4-, 5-, and 6-inch cubes are desirable but not necessary.

Suggestive Problems in Linear Measure

Find a 7-inch measure.

Draw a line 11 inches long.

Find a 7-inch measure; find a 3-inch measure. Build.

Draw a line 13 inches long. Mark off 8 inches.

Find a measure 11 inches long. Find one 9 inches shorter.

Compare a 14-inch measure with an 11-inch measure.

Find one measure as long as a 7-inch measure and a 5-inch measure together.

Build a measure 1 foot long with two measures not alike.

Find how many times you must use a 10-inch measure in measuring the length of the blackboard.

Find five 3-inch measures. Build.

Draw a line four times as long as a 5-inch measure.

Draw a line 2 feet 3 inches long. Show the 9-inch lines in it.

Draw a line 7 inches long. Call it a *one*. Draw the *three*.

A line 1 yard long is the *four*. Find the *one*.

Find two measures, one of them one-half as long as the other.

Suggestive Problems in Surface Measure

Find a tablet containing 24 square inches.

Build a square containing 16 square inches.

Find three tablets not alike each containing 36 square inches.

Count the perimeter of a surface containing 18 square inches.

Count the perimeter of a surface 7 inches long and 3 inches wide.

Draw a line 1 foot long. Find a tablet with a perimeter as long as this line.

Build a tablet containing 24 square inches with two equal tablets.

Find a tablet 8 inches long and 1 inch wide; find one just as long but twice as wide. Build.

Find a tablet containing 10 square inches. Think what you must put with it to make a tablet containing 18 square inches. Build.

Show four equal surfaces in a surface containing 28 square inches.

Show the 2-inch squares in a surface 10 inches long and 2 inches wide.

Find a tablet containing six square inches. Call it a *one*. Build the *seven*.

A surface 1 yard long and 1 inch wide is the *twelve*. Find the *one*.

Find two tablets one six times as large as the other.

Enclose a rectangle containing 16 square inches. Enclose another one-half as large.

Draw a square foot. Show the 4-inch squares in it.

Draw to a scale a plan of the top of a table 8 feet long and 3 feet wide.

Suggestive Problems in Solid Measure

Build a solid 3 inches long, 2 inches wide, 2 inches thick.

Find a solid containing 16 cubic inches.

Using three equal solids, build one containing 18 cubic inches.

Show four equal solids in one 4 inches long, 2 inches wide, and 2 inches thick.

Find three different solids each containing 8 cubic inches.

Find a solid with six square faces. Count the number of square inches in the entire surface.

Find a solid with four faces alike.

Find a solid with only two faces alike. Count the area of the entire surface.

Draw the faces of a solid 4 inches long, 2 inches wide, 1 inch thick.

Lay with tablets the faces of a solid 5 inches long, 1 inch wide, 1 inch thick.

Find five solids each containing 3 cubic inches. Build.

Find two solids one of them one-third as large as the other.

A solid containing 4 cubic inches is the *one*. Build the *five*.

A 3-inch cube is the *nine*. Find the *one*.

A Set of Problems for Grade III., using Six as a Number Measure

Draw a line 6 inches long. Draw one 7 inches long. Draw one as long as both together.

Find a 6-inch measure. Draw a line 9 inches longer than this measure.

Find a measure 6 inches longer than a 4-inch measure.

Build with two solids a block containing 14 cubic inches. Use one solid containing 6 cubic inches.

Find a tablet 5 inches long and 2 inches wide. Find one 3 inches long and 2 inches wide. Build.

Find the measures 1 to 3 inches long. Draw lines 6 inches longer than each of these measures.

Build measures 6 inches long in as many different ways as you can.

Find two different tablets each containing 6 square inches.

Find two different solids each containing 6 cubic inches.

Count the perimeter and area of a 6-inch square.

Find a tablet whose length is 6 inches longer than its width.

Find a tablet whose width is 6 inches and whose length is 5 inches longer than its width.

Find a solid whose entire surface contains 6 square inches.

Build a solid 3 inches long, 2 inches wide, 1 inch thick. Count the entire surface beginning with the largest faces.

Draw a line 1 foot and one-half long. Show the 6-inch lines in it.

Enclose a rectangle containing 15 square inches. Show two unequal rectangles in it, one containing 6 square inches.

Draw a line one-half as long as a foot-rule.

Find measures 7, 8, 9, and 10 inches long. Find measures 6 inches shorter than each of these.

Draw a line 2 feet long. Call it a *four*. Show the *ones* in it.

Show tablets containing 6 square inches in a tablet 6 inches long and 5 inches wide.

Find an 11-inch measure; find a 6-inch measure. Draw a line as long as the difference between these measures.

Draw a line 13 inches long. Show two lines in it, one 1 inch longer than the other.

Find two tablets one six times as large as the other.

Find two solids one six times as large as the other.

Find a tablet 5 inches long and 1 inch wide. Find one six times as large.

A tablet 7 inches long and 1 inch wide is the *one*. Find the *six*.

A 2-inch square is the *one*. Find the *six*.

Draw a line six times as long as a 3-inch line.

Count the area and perimeter of a tablet 8 inches long and 6 inches wide.

Find a tablet containing 10 square inches. Find one six times as large.

Find the *six* for a 3-inch square.

Build a solid containing 48 cubic inches, that is 6 inches long.

Show six equal solids in a solid containing 24 cubic inches.

Find six equal tablets each containing 2 square inches. Build.

Draw a line 6 inches shorter than a foot-rule.

Find a tablet 10 inches long having its width 4 inches shorter than its length.

Draw a line 2 inches longer than a foot-rule. Mark off 8 inches.

Count the perimeter of a tablet containing 42 square inches, 7 inches long.

Show the 6-inch lengths in a measure 1 foot long.

Find a measure 7 inches shorter than a 13-inch measure.

A tablet containing 6 square inches is the *one*. Find the *eleven*.

Count the area of a surface 6 inches wide and twice as long as it is wide.

A line 1 yard long is the *six*. Find the *one*.

Find a tablet containing 60 square inches. Find another one-tenth as large.

Show the 3-inch squares in a surface containing 54 square inches.

Cut off a string 1 foot 4 inches long. Cut another 10 inches shorter.

Draw a line 2 yards long. Draw another one-sixth as long.

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